When measuring the H1-line with the Embrace system, there is a noticable ripple in the frequency response. This effect has been proven somewhat difficult to calibrate at first. Later it was discovered that the issue was caused by aliasing. This is a short summary of the derivations performed to remove the filter ripple.

Conceptually, the polyphase filter essentially consists of a FIR filter, followed by down-sampling.

The FIR filter is implemented as a convolution of the input signal with the impulse response of the filter. In the spectral domain the convolution translated to a multiplication of the input signal (s) with the Fourier transform of the impulse response (f). The filtered output (p) can therefore be written as

$$p(\omega) = s(\omega) \cdot f(\omega) \tag{1}$$

The filter should limit the frequency range, to allow down-sampling to take place. However in case of the Lofar back-end, the filter bandwidth is more than the Nyquist bandwidth of the decimated output, so aliasing effects occur.

The aliasing causes the spectral content of all other Nyquist zones to 'fold back' to the first Nyquist zone. Mathematically this effect can be described by a convolution of the spectrum with a Dirac comb. The output of the down-sampling is given by

$$d(\omega) = p(\omega) * \Delta(\omega) = [s(\omega) \cdot f(\omega)] * \Delta(\omega) \tag{2}$$

where $*$ denotes convolution and $\Delta(\omega)$ is the Dirac comb function, defined as

$$\Delta(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n \cdot \omega_{\text{subband}}) \tag{3}$$

where δ is the Dirac delta function and ω_{subband} is the bandwidth of a Lofar sub-band, which is the sample frequency divided by the decimation factor (200 MHz / 1024).

To calculate the power each frequency, we need to determine the expected value of the square of the magnitude.

$$E\left[|d(\omega)|^2\right] = E\left\{|[s(\omega) \cdot f(\omega)] * \Delta(\omega)|^2\right\} \tag{4}$$

It is assumed that no correlation exists between the aliased spectral components, therefore, the expected value of the magnitude of the sum squared is equal to the sum of the magnitudes squared.

$$E\left[|d(\omega)|^2\right] = E\left(|s(\omega)|^2\right) * \left[|f(\omega)|^2 \cdot \Delta(\omega)|^2\right] \tag{5}$$

So, the measured output spectrum is a convolution with the response of the input filter multiplied by a Dirac comb function.

The retrieve the original spectral content from the output data, is a matter of undoing this convolution step, by any kind of deconvolution algorithm. Currently this deconvolution is based on Fourier transforms, but there is ample opportunity to devise better algorithms.